LEGAL INSIDER TRADING, CEO'S INCENTIVES, 
AND QUALITY OF EARNINGS

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ABSTRACT

The recent accounting scandals brought into light the failure of corporate governance mechanisms to curbing earnings management. This study focuses on the insiders who design the managers' compensation contracts. The contract designers are seen as lacking the financial expertise to correctly uncover the true outcome. However by virtue of their knowledge of the contract details, they can discern the likelihood that the firm’s public report is not truthful. Modeling the firm as a principal-agent contract, we show that insiders induce earnings management and make trading gains by designing suboptimal incentives. Given that our results are driven largely by the lack of these directors' financial expertise, our study has the policy implication that inclusion of financial experts in compensation committees can contribute to transparencies under the current insider trading rules in place.

Keywords: legal insider trading, quality of earnings, principal-agent contract, bluffing

1. Introduction

Firms are not required to disclose the precise details of their managers’ compensation (the compensation function: the exact specification of how pay is determined, what factors affect it and how do these factors affect the pay level). Since accounting earnings may be managed to increase bonuses, an opportunity arises for profitable trading by non-executive insiders such as board or compensation committee members who are knowledgeable about the relationship between the publicized accounting reports (earnings) and the true earnings. Profitable insider trading is thus made possible because of the insiders’ superior ability to interpret the public financial reports. Presumably, this superior ability is gained as a result of knowing the details of the compensation contracts and the implication thereof for the incentives to manage earnings.

Seyhun (1998) writes:
If insiders cannot trade on corporate announcements, what sort of information can they trade on? Insiders can clearly trade on the basis of their understanding and interpretation of public information outside the moratorium periods. For instance, assume that the stock price of the firm goes down sharply. The decline of stock price is, after all, public information. Now suppose that insiders do not know anything about their firm that would justify such a price decline. Insiders in this case can comfortably buy stock of their firm (and support the market) without worrying about insider-trading regulations.

In this study, we examine the relationship between earnings management and legal insider trading by insiders who know the manager’s compensation contract, but otherwise lack the expertise or ability to know the magnitude of the true earnings. We analyze the effect of the motif and opportunity for such insider trading on the firm’s value and the quality of its accounting earnings.

Modeling the firm as a principal-agent contract with an unobservable outcome, we find that insider trading affects the shape of the contract and hence the firm’s value and quality of earnings. Specifically, the optimal compensation schedule first increases and then levels off (see Figure 2 below). The increasing part has a steeper slope than that of a standard principal-agent contract. The cap (upper bound)—the report that maximizes the manager’s compensation -- becomes the manager’s target report. That is, the manager will attempt to manage earnings by reporting
the target outcome.

The contract incentivizes the manager, at a cost. The incentive effect is created by imposing risk on the risk-averse, work-averse manager. It induces the manager to exert a higher level of effort, which increases the firm’s gross expected value. The cost is twofold: one is that the more risky contract necessitates the payment of a premium to the risk-averse manager, thus decreasing the shareholder’s residual share. The other is the compromised transparency of the accounting earnings.

Transparency is compromised because the manager attempts to inflate the report, and because the target report is known only to insiders. Suspecting earnings management, the market adjusts downward the inferred outcome even when the true outcome is reported. This weakens the association between stock price and reported earning. To sharpen this point, consider, by contrast, the case in which the market knows the magnitude of the target outcome. In this case, the market will adjust downward only the target report (outcome) and accept as truthful any other report.

Despite the extensive research on governance, there is only a handful of analytical studies in this area (see the literature review by Becht, Bolton, and Roell, 2002, and the citations in Ronen, Tzur, and Yaari, 2006), most are unconcerned with the link between governance and transparency of earnings.2 This paper is part of our recent research effort directed at understanding why earnings management3 has prevailed the US capital markets (Ronen and Yaari, 2006, and Ronen, Tzur, and Yaari, 2006). In Ronen and Yaari, 2006, we are concerned with the question of why the board of directors does not design truth-inducing contracts. The answer there is that the limited-liability of the manager might render such contracts too costly. In this study, however, we have unlimited liability; rather, directors choose to not design the most efficient contract, because they can gain from reduced transparency of the reports. In Ronen, Tzur, and Yaari, 2006, we show that rational shareholders should provide the board of directors with incentives to design an efficient contract with management, and that such incentives, in turn, induce the directors to collude with management in earnings management, in order to make insider-trading gains on the firm’s stock they hold by virtue of their incentives package. There are a few differences between the above paper and the current study: First, in this study, we focus on two players only, ignoring the conflict of interests between shareholders and their delegates -- the board of directors. Second, In this paper, we are concerned mainly with compensation committee members who lack the financial expertise uncover the true earnings, but who by the virtue of designing management’s incentives, are able to infer the management’s target report. Hence, their insider trading activity is not illegal. In this regard, our findings suggest that the inclusion in the compensation committee of financial experts (who can uncover the true outcome) can improve transparency in the presence of extant insider trading rules. Third, this study characterizes the social value of the firm and price distortions.

The paper is organized as follows. In Section 2, we present the model. Section 3 contains a characterization of the optimal contract, and in Section 4 we analyze the quality of the accounting earnings. We conclude in Section 5. The proofs are relegated to appendices.

2 There exists an accounting literature that studies earnings management in principal-agent relationships (see Ronen and Yaari, 2007). For example, an earlier paper that links the disclosure of contracts to earnings management is Dye, 1988. Using an overlapping generations model, he shows that when contracts are not public knowledge, each generation of sellers induces the manager to manage earnings to increase the price obtained from the next generation -- the buyers. His results are based on ‘signal jamming’ dynamics. Since outsiders are unable to undo the manager’s report to discover the firm’s true value, they (correctly) postulate that the report is prepared with intent to inflate the price, and they respond by discounting it. Insiders respond by inflating the report (see Chapter 1 in Ronen and Yaari, 2007). Note that this type of earnings manipulation has no effect on the transparency of earnings if the discount is estimated correctly by outsiders). This study differs from Dye’s in two regards. First, Dye restricts his analysis to the case in which the contract designers sell their shares (for a further discussion of this point, consult Demski, 1998). Thus, it is obvious that shareholders would like to manage earnings to maximize the expected price. In our study, owners can either sell (where an inflated price is preferable) or buy (where a deflated price is preferable). Second, we inject misrepresentation differently, allowing for some reports to be truthful (see details below), which, in turn, affects the shape of the contracts.

3 Earnings management is defined as “the practice of distorting the true financial performance of the company. [SECURITIES AND EXCHANGE COMMISSION 17 CFR Parts 210, 228, 229, and 240. Release No. 34-42266J].

4 As Scott (1997) and others have noted, earnings management refers to a plethora of strategies. The study of the firm as a one-shot game implies that the earnings management strategy is maximization (overstating income). In a multi-period horizon, the manager may engage in income smoothing (see, e.g., Sankar (1999)) or in taking a bath (Healy (1985)). We discuss these issues below, but, in general, our modeling of the effect of earnings management cum insiders’ trading on the quality of accounting earnings in a one-shot game does not affect the results qualitatively.
density function of outcome \( f(x|a,e) \), with the associated cumulative distribution function, \( F(x|a,e) \), with support \([0,X]\) (the assumption that minimum earnings are zero is innocuous).

Thereafter, the insiders design the risk-averse, work-averse manager's compensation schedule \( S \), basing it on the imperfectly audited, end-of-the-period report, \( m \), \( S: m \rightarrow \mathcal{M} \). The contract is private information between insiders and the manager. In what follows, we denote the report that rewards the manager with the maximum payment, \( s_{\text{max}} \), by \( L \). (\( L \) might be neither unique nor different from the truth.)

The manager then makes unobservable production-investment decisions that require effort, \( a \).

At the end of the period, nature chooses the general parameter, and the actual outcome, \( x \), \( x \geq 0 \), is realized (the assumption of profitable outcomes eases presentation only and is thus innocuous). The manager alone observes \( x \) and communicates an outcome that may or may not equal \( x \) to the firm’s auditor. The manager may inflate reported earnings but whether he succeeds to misrepresent depends on factors beyond his control, such as reversal of accruals from transactions made in the past (Ronen and Yaari, 2007, chapter 9); the willingness of suppliers and customers to cooperate with him; and the probability that an imperfect audit discovers the truth. In what follows, we denote the probability of successfully inflating the report by \( \beta \), \( \frac{1}{2} < \beta < 1 \).

After publicizing the audited report, \( m \), the manager receives compensation, \( s = S(m) \). The market price, \( P \), is set, and insiders may trade in the firm's shares. We assume that insiders lack the financial expertise necessary to infer the true outcome, \( x \), from the observable \( m \). So, effectively, the situation is similar to Demski and Sappington’s, 1987, wherein lack of expertise prevents the less informed party from eliciting the truth from the better-informed one – a blocked communication scenario that renders the revelation principle inapplicable. The market price might be different from the insiders’ valuation, \( P' \).

A new period begins, and after awhile the market receives additional information that eliminates the gap between the market’s and the insiders' evaluations. This assumption guarantees that insider trading is profitable.

Figure 1 summarizes the time-line.
EV(x)]. If insiders believe that the price is too low, they instead buy additional \( W/P(m) \) shares when the market opens after the public information report and earn \( W/P(m)[EV(x) - P(m)] \). To ease notation, in what follows, we normalize \( N \) to 1 (the interpretation of this normalization is that all the arguments in the insiders’ program are expressed as per share held by insiders).

(v) The risk-averse, effort-averse manager maximizes a von Neumann-Morgenstern utility function that is separable in monetary reward and effort, \( U(s)a \), where \( U'>0 \), \( U''<0 \), and \( e>0 \). The manager can obtain utility of \( \bar{U} \) by being employed in an alternative job. We assume that the manager is ethical, in that when a truthful message yields \( s_{\text{max}} \), the manager strictly prefers to report the truth.

This characterization implies that the manager’s preferences are lexicographic. His payoff over compensation and effort takes priority over his preference to report the truth.

(vi) The assumptions on technology are the following:
(a) To ensure regularity, we assume that all functions are twice continuously differentiable.
(b) The following standard assumptions hold:
1. The Monotone Likelihood Ratio Condition (MLRC) holds; i.e., \( f_{x}/f_{y} \) is an increasing function of \( x \), where \( f_{x} \) is the derivative of \( f(x|a,e) \) with respect to effort.
2. The first-order approach is valid [either the CDF condition holds; i.e., \( F(x|a,e) \) is convex in effort (Rogerson 1985), or the conditions of Jewitt's (1988) Theorem 1 hold]
3. The support of \( f(x|a,e) \) is independent of effort.
(c) To avoid complicating the analysis with boundary conditions, we assume that when \( m=0 \), \( S(0)=s_{0} \), and without loss of generality, we set \( s_{0}=0 \).

3. The equilibrium

Denote by \( E_{z} \) the expectation when the beliefs’ function is \( z \), \( z=f(a,h) \). Table 1 summarizes the key elements of the model, which includes strategic players—the manager and the insiders—as well as outsiders. Publicly, all observe the firm’s report, \( m \), the outsiders’ prior beliefs, \( h(x) \), and the manager’s realized compensation, \( x \), and all know the accuracy of the audit, \( s_{m} \) and the marginal density function of the firm-specific parameter, \( g(e) \).

Table 1. Summary of key features of the game (a comparison of Table 1 with Table 2 in our companion paper (Ronen, Tzur, and Yaari, 2006), shows that the games are different regarding players, decisions, and payoffs)

<table>
<thead>
<tr>
<th>Private Information</th>
<th>Strategy</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manager</td>
<td>( a,x,f(x</td>
<td>a,e) )</td>
</tr>
<tr>
<td>Insiders</td>
<td>( f(x</td>
<td>a,e) )</td>
</tr>
<tr>
<td>Outsiders</td>
<td>none</td>
<td>( P(M) )</td>
</tr>
</tbody>
</table>

Note that we assume away insider trading by the manager. This assumption is innocuous because corporate articles allow insiders a given window of trading. Since managers and other insiders are likely to trade at the same time, the impact of the disclosure of management’s trading (within two days after the trading) on the price takes place after the insiders’ trade. The impact of the trade on the price is negligible if the market-price setter cannot identify the trader as management. We characterize the sequential rational equilibrium: The players choose the strategy that maximizes their payoffs in the remainder of the game, given their beliefs, and their beliefs are consistent with the strategies through Bayes’ rule when it is applicable. Specifically,

(a) The manager chooses effort, \( a \), that maximizes his expected utility and, conditional on the realized outcome, \( x \), he chooses the report, \( m \), that maximizes his expected bonus.
(b) The insiders design the contract by solving an optimization program given their beliefs, \( f(x|a,e) \). (See details in Section 3.1.) Their sell/buy strategy depends on their valuation of the firm relative to the market.

c) Outsiders update their prior beliefs on the outcome to \( h_{P}^{P}(x|m) \) after observing the report, \( m \), through Bayes’ rule given the strategies of the manager and insiders.
3.1. The Optimal Contract
At the beginning of the period, the insiders design the contract by solving the following program:

\[
\max_{s} (1 - \beta) E_{s} V(x - S(M(x))) + EG \quad \text{s.t.} \\
E_{s} U(S(M(x))) - a \geq U \quad \text{(PC)} \\
a = \arg \max_{a \in A} \{E_{s} U(S(M(x))) - a\} \quad \text{(IC.a)} \\
m = \arg \max_{m \in [0, X]} \{(1 - \pi)U(S(m|x)) + \pi U(S(x) - a)\} \quad \text{(IC.m)}
\]

The insiders maximize their expected payoffs subject to three constraints. The first, (PC), states that the manager is willing to participate in the contract because it guarantees him his reservation utility, \( \overline{U} \). The second and third, (IC.a) and (IC.m), are the incentive constraint with respect to effort and report, respectively.

**Proposition 1**

(a) The optimal contract is a non-monotonic function. For every \( e \), there is a report \( \hat{L}(e) \), such that the compensation schedule increases up to \( \hat{L}(e) \), \( S' > 0 \) for \( m < \hat{L}(e) \), and then flattens out, \( S' = 0 \) for \( m \geq \hat{L}(e) \).

(b) If the outcome falls in the increasing part, \( x < \hat{L}(e) \), the manager inflates the report, \( M(x|e, x < \hat{L}(e)) = \hat{L}(e) \). If \( x > \hat{L}(e) \), the manager reports the truth, \( M(x|e, x > \hat{L}(e)) = x \).

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**Figure 2.** The Compensation Schedule

The proof is based on the solution of the insiders' program. We sketch the proof of Part (a) here. [The proof of Part (b) is immediate from Part (a).] First, we partition the set of all outcomes into two types of compact sets: sets of reports that yield \( s_{\text{max}} \), and compact subsets, which award the manager less than \( s_{\text{max}} \) if he reports the truth. For example, the following hypothetical schedule has two subsets of the first type (the heavy lines) and three subsets of the second type.

**Figure 3.** A Hypothetical Contract
We show that the incentive contract must be an increasing function in the intervals where \( S \) does not reach a maximum. Hence, it is impossible to have a continuous contract with declining segments as in Figure 3. Moreover, the set of maximum payments must lie to the right of the subset of payments that are less than the maximum. That is, the manager’s compensation schedule first increases; and at some point \( \hat{L}(e) \), it flattens out.\(^5\) The contract is not a strictly increasing function because of trading gains. If \( \hat{L}(e) = \bar{X} \), insiders cannot make trading gains when the firm reports a truth that is lower than \( X \), because every investor believes the report. To see this point, suppose that outsiders believed that \( X \) is the target report of the manager, \( \hat{L} = \bar{X} \). Then, insiders can make speculative gains by shifting \( \hat{L} \) to the left by \( \tilde{\varepsilon} \) they thereby gain an information advantage: when the firm reports \( \hat{L} \), \( \hat{L} = \bar{X} \), they alone know that the report is likely to have been managed and hence ignore it while the market believes it, and when the report is \( X \), they alone know that this is the truth, \( x = \bar{X} \), valuing the firm at \( \mathcal{V}(X-S(m)) \), while the outsiders adjust the value downward.

This schedule implies that the manager tries to report \( m = \hat{L}(e) \) when the actual outcome, \( x \), is lower than \( \hat{L}(e) \), and the truth when the outcome, \( x \), exceeds \( \hat{L}(e) \). The next proposition analyzes how insider trading affects the shape of the contract for \( x < \hat{L}(e) \).

**Proposition 2:**
Denote by \( S_0 \) the sharing rule when \( \beta = 0 \) and the insiders are not interested in making trading gains. Then, \( S' > S_0 \) for \( x \leq \hat{L}(e) \).

**Proof:** See Appendix.

The compensation schedule is an increasing function up to \( \hat{L}(e) \). Proposition 2 shows that, because the contract includes a flat region, the increasing part must be steeper to induce the manager to exert effort.

The next proposition investigates the incentives effect of the contracts.

**Proposition 3:**
(a) The contract that is motivated by trading gains induces a higher level of effort.
(b) The contract induced by trading gains increases the gross expected value of the firm, \( E(x) \).
(c) Although the contract designed to generate trading gains increases the firm’s expected value, \( E_x \), the shareholders’ residual receipts are lower than when insiders design \( S_0 \).

Comparison of our contract with one that is not motivated by trading gains shows that the former is more risky for outcomes below \( \hat{L}(e) \), and that this increased riskiness increases the effort exerted by the manager. Since greater managerial effort increases the expected profits, the trading gains might increase social welfare. Since a piece-wise contract is feasible when trading gains do not play a role, the increase in the firm’s value does not inure to the benefit of the shareholders: had this been the case, the optimal \( S_0 \) contract would have been this piece-wise contract.

Note that although capped compensation contracts are a well-documented phenomenon, they present a puzzle, because the principal-agent paradigm predicts a monotone-increasing sharing rule (Holmstrom (1979), Harris and Raviv (1979), and others). Propositions 1 and 3 explain this phenomenon as the equilibrium outcome of the principal-agent relationship between the firm’s manager and insiders, when the latter maximize trading gains.

### 4. The Quality of Accounting Earnings

It is commonly understood that the quality of accounting earnings is measured by their transparency in revealing the underlying economic earnings. Our results indicate that this definition encompasses two requirements: the firm reports the truth, and everyone believes the report. In what follows, we measure the quality of accounting earnings by the likelihood of making trading gains, \( \text{Prob}[G] \). Clearly, if the firm reports the truth for all outcomes and everyone realizes this is the case, \( \text{Prob}[G] = 0 \).

To characterize the market’s beliefs, we distinguish between the set of firm-specific variables \( e \) for which the firm’s report could be “the optimal report,” \( \hat{L}(e) \), \( \{e | m = \hat{L}(e) \} \) with positive probability, and the set of firm-specific variables for which the report must be truthful, \( E(x) \).

The market’s beliefs are as follows:
If \( \text{Pr}(m = L) = 0 \), \( E(x) = \mathcal{V}(m-S(m)) \).

If \( \text{Pr}(m = L) \neq 0 \),
\[ E(x) = \mathcal{V}(m-S(m)) \frac{1}{\text{Pr}(m = L)} \int_{x}^{\bar{L}(e)} g(e) \, de + \int_{x}^{\bar{L}(e)} g(e) \, de \left(1 - \frac{1}{\text{Pr}(m = L)} \int_{x}^{\bar{L}(e)} \frac{1}{E(x)} \mathcal{V}(m-S(m)) \frac{1}{H(m,S(m))} \, de \right) \left(1 - \frac{1}{\text{Pr}(m = L)} \int_{x}^{\bar{L}(e)} \mathcal{V}(m-S(m)) \frac{1}{H(m,S(m))} \, de \right) < \mathcal{V}(m-S(m)). \]

The outsiders’ posterior evaluation is a Bayesian update of the prior. If they believe that there is no firm-specific variable that yields the observable \( m \) as
the manager’s target optimal report, they believe that the report is truthful. Otherwise, they weight the probability that the report is truthful against the probability that it is the optimal report and the firm’s true value is lower.

Since the firm’s reporting strategy is to overstate low outcomes, \( x < \hat{L} \), and report the truth for higher outcomes, 
\[ M(x) = \begin{cases} \hat{L} & \text{if } x \leq \hat{L}, \\ x & \text{if } x > \hat{L}. \end{cases} \]

The insiders know that the firm reports the truth whenever \( m > \hat{L} \), then the firm reports the truth because the manager does not wish to misrepresent. If \( m < \hat{L} \), the firm reports the truth because the auditor detected the truth. When \( m = \hat{L} \), the insiders know that the event of the firm reporting the truth has a probability of measure zero, so the firm must be misrepresenting its lower earnings.

The Wierstrasse-Erdmann condition (see Hadley and Kemp (1971)) implies that \( G(x) = G(\hat{L}(e)) \). \( G(x) \) is the trading gain when the firm reports the truth, and \( G(\hat{L}(e)) \) is the trading gain for successfully reporting \( \hat{L}(e) \). This condition, together with Equations (2) and (3), implies that when the market is unsure about the report, 
\[ V(m - S(m)) > P^m(m) > P^0(m). \]

The implications for the quality of earnings and the market price are given in Proposition 4.

Proposition 4:
(a) In expectations over \( h(x) \), when \( m = \hat{L}(e) \), the market price overstates the firm’s value, and insiders sell their shares. When \( m \neq \hat{L}(e) \), the market price understates the firm’s value, and insiders buy additional shares.
(b) Trading gains compromise the quality of the accounting earnings.

Proposition 4 describes the effect of the accounting earnings on the quality of the market price. When \( m = \hat{L} \), insiders, knowing that the report misrepresents the outcome, evaluate the firm at a lower price than outsiders, who are putting some weight on the event that the firm has reported the truth. When \( m \neq \hat{L} \), insiders know that the firm reports the truth, and their expected evaluation, 
\[ P^0(m | m = x), \]

is higher than the market’s expected evaluation, \( P^m \), since the market discounts the report.

The proof is immediate from the fact that if \( \hat{L}(e) \) is publicly known, whenever \( m \neq \hat{L} \), insiders and outsiders know that the firm reports the truth, so that \( G(x) = 0 \) for all truthful reports. Now, suppose that the insiders design \( \hat{L}(e) < X \). Since the outsiders are rational, they can infer the firm-specific parameter, \( e \), and the information asymmetry between the insiders and the outsiders disappears.

5. Conclusion

The law defines corporate insiders as officers (managers), directors, and beneficial owners. We analyze the effect of disclosure of the parameters of the manager’s contract on the firm’s value and market price through its effect on insiders’ ability to make trading gains.

We study insider trading in a model that includes insiders, outsiders, the manager, and noise traders. Familiarity with the details of the manager’s contract provides insiders with an insight into the manager’s choice of the reported outcome relative to the realized outcome. Insiders achieve an information advantage by designing a capped contract (the manager’s compensation does not increase beyond a critical level), in contrast to the monotone-increasing schedule that is predicted by the standard principal-agent paradigm. The equilibrium is determined by a trade-off between a completely increasing compensation schedule, which would generate managerial incentives but induce misrepresentation, and a completely flat schedule, which would always endow insiders with perfect information but would diminish any managerial incentive to exert effort.

As insiders act upon public information and their information is accurate only when the firm reports the truth, outsiders cannot win a lawsuit against insiders, even if their private information was verifiable. Makers of accounting rules, however, can reduce the scope of this trading by requiring disclosure of the target report.

References

The realized gain per share at the end of the period, $G$, is given by:

$$G = \frac{\text{Sale Price per Share} \times \text{Number of Shares Outstanding} - \text{Purchase Price per Share} \times \text{Number of Shares Outstanding}}{\text{Number of Shares Outstanding}}$$

## Appendix

We start with a general compensation function. Since the manager is not paid more than some maximum level, $s_{\text{max}}$, we partition the interval $[0,\bar{X}]$ of possible messages into two types of connected subsets: the manager is either paid less than $s_{\text{max}}$, $m \in \tilde{m}_i = \{ m \mid m \in (x_i, x_{i+1}) \text{ and } S(m) < s_{\text{max}} \}$, $i=1,2,3,\ldots$, or, the maximum, $s_{\text{max}}$, $m \in \tilde{m}_k = \{ m \mid m \in [x_k, x_{k+1}] \text{ and } S(m) = s_{\text{max}} \}$, $k=1,2,3,\ldots$ That is, $\tilde{m}_i \cup \tilde{m}_k \subseteq [0,\bar{X}]$, and for $i,k$, $\tilde{m}_i \cap \tilde{m}_k = \emptyset$.

The thrust of the proof is that there is only one subset of each type and that because the compensation schedule is a strictly increasing, strictly continuous function on $\tilde{m}_i$, it must be the case that $\tilde{m}_i$ lies to the left of $\tilde{m}_k$.

### The Speculative Gains

The realized gain per share at the end of the period, $G$, is given by:

$$G = \frac{\text{Sale Price per Share} \times \text{Number of Shares Outstanding} - \text{Purchase Price per Share} \times \text{Number of Shares Outstanding}}{\text{Number of Shares Outstanding}}$$
The valuation reflects Insiders' understanding that the message is truthful because the manager earns less than $s_{\max}$, while (A3) reflects their understanding that the message yields the maximum payment, and hence may misrepresent the truth. We allow for $f(x|m)$ to be degenerate since the optimal misrepresentation might coincide with the truth.

**Characterization of the contract on $\hat{m}_i$**

**Preliminary:**

Denote by $*\pi$ the equilibrium contract. Define a perturbation of $S^*$ on an arbitrary $\hat{m}_i$ as follows:

$$S = \left\{ \begin{array}{ll} S = S^* & \forall m \notin \hat{m}_i \\ S_i = S^*_i + \gamma q(.) & \text{where } q(x) = q(x_{\gamma}) = 0, \forall m \in \hat{m}_i \end{array} \right\},$$

To find the optimal compensation, we derive with respect to $\gamma$, set the derivative to zero and evaluate it at $\gamma = 0$.

Denote by $\hat{f}(x|m)$ the posterior beliefs function of outsiders, and by $D$ the dummy variable that takes the value of 1 when Insiders sell, Insiders solve the following program:

$$\begin{align*}
&\pi x_{i+1} \left[ V(x - S^*(x) - \gamma q(x)) + f(x|m, x) dx \right] + \\
&\pi (1-D) x_{i+1} \left[ V(x - S^*(x) - \gamma q(x)) + f(x|m, x) dx \right] + \\
&\max_{\gamma} \left\{ \beta(1-D) \pi \right\} x_{i+1} \left[ V(x - S^*(x) - \gamma q(x)) - \int_0^x V(x - S^*(x) - \gamma q(x)) h'(x|m, m = x) dx \right] + \\
&\lambda \pi x_{i+1} \left[ U(S^*(x) + \gamma q(x)) + f_i(x|m, x) dx \right] + \mu \pi x_{i+1} \left[ U(S^*(x) + \gamma q(x)) + f_i(x|m, x) dx \right]
\end{align*}$$

The first argument is the insiders' maximization of the value of the firm. The next two expressions are the speculative gains for buying and selling. The final two expressions are the (PC) and (IC) constraints, respectively. Note that (IC) is incorporated into the program, since this subset is reached only upon discovery of the truth.

Taking the derivative with respect to $\gamma$, evaluating it at $\gamma = 0$, and setting it to zero, yields the following pointwise equilibrium conditions:

$$\forall \ x \in \hat{m}_i,$$

$$\begin{align*}
&\pi x_{i+1} \left[ V(x - S^*(x) - \gamma q(x)) + f(x|m, x) dx \right] + \\
&\pi (1-D) x_{i+1} \left[ V(x - S^*(x) - \gamma q(x)) + f(x|m, x) dx \right] + \\
&\max_{\gamma} \left\{ \beta(1-D) \pi \right\} x_{i+1} \left[ V(x - S^*(x) - \gamma q(x)) - \int_0^x V(x - S^*(x) - \gamma q(x)) h'(x|m, m = x) dx \right] + \\
&\lambda x_{i+1} \left[ U(S^*(x) + \gamma q(x)) + f_i(x|m, x) dx \right] + \mu x_{i+1} \left[ U(S^*(x) + \gamma q(x)) + f_i(x|m, x) dx \right]
\end{align*}$$

Suppose, by contradiction, that $S$ is a decreasing function. Then, the left-hand-side (l.h.s) of (A5) is a decreasing function of $x$ (Note that the derivative of $P^O(x) - P^M(x)$ with respect to $x$ is:

$$\frac{d}{dx} \left( P^O(x) - P^M(x) \right) = - \frac{2 \sum \left( P^O(x)^2 - P^M(x)^2 \right) \frac{\partial P(x)}{\partial x} \ dx}{P(x)^2}.$$  

This inequality obtains because owners buy when $P^O(x) - P^M(x) > 0$, and by assumption of the proof $\frac{dS}{dx} < 0$. [Note: the derivative of $P^O(x) - P^M(x)$ with respect to $x$, is zero].

Since $S(.)$ is a continuous function over a closed interval, all sets of maximum payments must be to the right of the sets with lower payments or else, the compensation would be a decreasing function, which contradicts our result that the compensation is a non-decreasing function. Consequently, all subsets of messages that yield payment lower than the
maximum belong to one subset. In this case \( S(n) \) is a piecewise function; it increases up to a cap, \( \hat{L} \), and then it flattens off.

When the true earning fall below \( \hat{L} \), \( x < \hat{L} \), the manager attempts to report \( \hat{L} \). If true earnings are higher, \( x > \hat{L} \), the manager cannot gain from misrepresenting so he will communicate the truth to the auditor and hence, the firm reports the truth. Denote by \( G(x) \) and \( G(L) \) the speculative gains when Insiders believe and do not believe the report, respectively. To prove that \( \hat{L} < X \), for some \( e \), note that the contract must satisfy the Erdmann-Wierestrasse continuity condition, which is:

\[
0 = G(x|x = \hat{L}) + G(L|L = \hat{L}) - G(x|x = \hat{L} + 1). \tag{A6}
\]

Rearranging, at \( \hat{L} \), \( G(x) = G(L) \). At \( m = X \), \( G(x) = 0 \), since none believes the report. By the asymmetry of information between Insiders and Outsiders, \( G(L) > 0 \). This yields the required contradiction. Q.E.D.

**Proof of Proposition 2:**

We compare our program with the following:

\[
\max_{S_0} \mathbb{E}(V(x, S_0(x)))
\]

\[\text{s.t.} \quad \mathbb{E}(U(S_0(a)) = \mathbb{E}(U(S(X))) + (1-\pi) \mathbb{E}(U(S_{\max}(a)) - a \geq \bar{U}). \tag{PC}\]

\[a \in \text{argmax} \mathbb{E}(U(S_0(a))). \tag{IC.a}\]

Denoting by \( \lambda \) and \( \mu \) are the Lagrange multipliers of (PC) and (IC.a), the associated Euler equation yields the following pointwise conditions:

\[
\forall x, \quad \frac{U'(x)}{U'(a)} = \lambda_0 + \mu_0 \frac{f_e(x|a,e)}{f(x|a,e)}. \tag{A7}\]

Since condition holds for all \( x \), this schedule is a strictly increasing function with no corners.

A comparison of (A5) and (A7) shows that either (a) \( \lambda_0 \) and \( \mu_0 \), or (b) \( \lambda_0 \) and \( \mu_0 \). Since \( S \) includes a flat region which makes it a better risk-sharing arrangement, Case (a) holds only if \( S \) is steeper for low values of outcomes. Case (b) is ruled out because the piecewise contract is feasible in the \( S_0 \) program, but (A7) holds pointwise. Q.E.D.

**Proof of Proposition 3:**

Denote the agent’s effort and contract when the principal only maximizes the expected value of residual outcome by \( a_{0} \) and \( S_0 \), and when he seeks speculative gains by \( a_L \), and \( S \), respectively.

The expected utility of the manager under \( S \) is:

\[
\pi \left[ U(S(x)) f(x|a_L,e) dx + (1-\pi) \left( \int_0^\hat{L} U(S_{\max}(x)) f(x|a_L,e) dx + \int_{\hat{L}}^X U(S_{\max})(x|a_L,e) dx - a_L \right) \right].
\]

Rearranging,

\[
\pi \int_0^\hat{L} \left( U(S(x)) - U(S_{\max}(x)) \right) f(x|a_L,e) dx + U(S_{\max}(x)) - a_L = 0. \tag{A8}\]

The corresponding (IC.a) is:

\[
\pi \left[ U(S(x)) - U(S_{\max}(x)) \right] f_e(x|a_L,e) dx - 1 = 0.
\]

Absent trading gains motive, the manager’s payoff is:

\[
\pi \left[ W(S_0(x)) f(x|a_0,e) dx + (1-\pi) W(S(X)) - a_0 \right].
\]

The corresponding (IC) is:

\[
\pi \int_0^X W(S_0(x)) f_e(x|a_0,e) dx - 1 = 0. \tag{A9}\]

The comparison of (A8) and (A9) shows that the argument that multiplies \( f_e \) is negative in (A8) and positive in (A9). Since both arguments equal \( 1 > 0 \), \( f_e \) must be more negative in (A8), which by the MLRC assumption, implies that \( a_L > a_0 \).

The proofs of parts (b) and (c) follow. A higher effort increased outcome in a first-stochastic-dominance sense, but since this contract is feasible when the contract is not induced by trading gains, the shareholders’ share must be lower. Q.E.D.

**Proof of Proposition 4:**

The proof is immediate from the discussion in the test. At the kink, the market trusts the report while insiders fully discount it; and at any other report, insiders alone know that it is the truth. Q.E.D.