PROFIT AND LOSS SHARING CONTRACTS AS A PRISONERS DILEMMA: AN AGENT BASED SIMULATION WITH GAME THEORY APPLICATION TO PARTICIPATIVE FINANCE

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Abstract

PLS contracts, Like Musharakah in participative finance, represent a practice of profit and loss sharing contracts. It is claimed to be a fair economic mode of investment as it entails the sharing, by the participants, of profits and risks. This mode of financing, however, suffers from asymmetric information in the form of adverse selection and moral hazards. In this Agent based simulation we managed to apply a repeated game theoretical approach to PLS financing using an agent based simulation tool called Netlogo. The purpose is to test whether PLS contracts are representative of a prisoner's dilemma game. We have identified different parameters which are used to calculate the payoffs of the bank and the enterprise which seeks financing. Each agent in this simulation has some strategies that he/she can use through the game. We have managed to run the simulation 1000 times for different model parameters under each combination of the agent's strategies. We have found evidence that PLS contracts are not representative of a prisoner dilemma game as mutual cooperation does not lead to a better payoff to the corporation than mutual defection. Over a repeated process, however, we found simulation evidence that the threat by the bank to apply an unforgiving strategy towards defection, leads to a cooperative behavior by the corporation through the strategy Tit-for-Tats.

Keywords: PLS Contracts, Agent-Based-Simulation, Moral Hazards, Prisoner’s Dilemma, Social Value, Adverse Selection, Nash Equilibrium, Netlogo

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1. INTRODUCTION

Profit and loss sharing contracts (PLS) generally suffer from asymmetric information in two ways. First, they suffer from adverse selection where the financier, in our case the bank, has difficulty in selecting an appropriate agent (in our case the corporation). Second, these contracts suffer from moral hazards where the agent, after being selected, has a tendency of misbehaving either through low effort or misreporting the results.

To dilute the effects of asymmetric information, many mechanisms have been proposed.

For example, collateral is one such mechanism applied by agents to signal their efficient type. This is in harmony with claims that collateral in debt contracts can be used to lower information asymmetries [1]. This is also in harmony with the proposition of Karim [2] with respect to submission of collateral in PLS contracts. This practice is, however, prohibited in Islamic PLS contracts but permissible in a conventional PLS system. The submission of a warranty by the financed corporation is permissible only if there is proof of negligence or non-respect of the contract terms from its part[3].

Another mechanism to reduce asymmetric information is the willingness of a confident manager to accept a low job protection. This is consistent with previous research as in [3]. Low job protection, however, can be seen as unfair to the entrepreneur since failure of the project can be due to factors beyond the entrepreneur's control [4]. Demanding security by the bank, in the form of low job protection, can be seen as making the entrepreneur lose more than his contribution[4]. This, however, contradicts the musharakah principle which calls for a fair sharing of profits and losses as mentioned earlier by Usmani [5].

A third mechanism proposed is Information sharing. Borrowers are shown to exert high efforts when their details are entered with credit bureaus [6]. This mechanism's impact is strong when borrowers’ mobility is higher [7] and if asymmetric information problems are more important [8]. It is problematic; however, that weaker banking competition can result from information sharing, [9].

PLS contracts suffer from misreporting project’s results by the financed agent compared to its conventional counterparts. Higher due diligence is a mechanism proposed to reduce this problem in Islamic PLS contracts [10].

Sharika shall be deemed to be trustees in respect of Sharika assets, however, as trustees they shall be jointly and severally liable for misconduct, negligence or breach of contract.
The unfair distribution of returns if the project fails can be a source of asymmetric information [11]. Because of the project’ risks, the financial institution may impose a higher profit sharing ratio. This however may result in lower projects return due to the financed agent lower motivation [11]. In our paper, charging a higher sharing ratio is referred to as defection from the bank. Defection also applies to the agent when he/she misreport the results. Another mechanism, to reduce information asymmetries in a profit and loss contract, is to have the agents contribute in the project’ capital [2]. In line with this finding, [12] proposes a minimal capital contribution by the financed agent and a minimal profit sharing ratio.

One research proposed that moral hazards can be solved under Mudaraba27 but not under musharakah (PLS) [13]. This can be criticized in a sense that under Mudaraba the bank provides the whole capital and therefore assumes all monetary risks. On the other hand under musharakah the capital is shared and intuitively the risk of losing capital is shared [4]. This is, also, inconsistent with our findings and the findings of Nabi [12] which proposes that moral hazards can be solved subject to a contribution from the entrepreneur, i.e moral hazard is more likely to be solved under a musharakah contract than under a Mudaraba contract. This is in line with the research of Inness [14] arguing that sharing contract is not feasible in case of total external financing of the project.

Another finding, proposed the usage of two profit sharing ratios instead of one to reflect the effort of the entrepreneur compared to the bank [15]. This model, however, does not treat the case of asymmetric information.

An incentive scheme was proposed in a preceding paper [16] to reduce asymmetric information. We have a game theoretical evidence that a higher social value and more flexibility in negotiating the profit sharing ratio and the capital contribution.

To evaluate PLS contracts in reducing moral hazards, We proposed a new model called ROMCA [17] and tried to assess it in relation to other forms of financing like debt finance and ROSCA. We found a simulation evidence that our model can dominate the other forms under adverse random shocks with low market conditions and prevailed in cases of moral hazards [17].

In order to help financial institution in their agent selection process, we have developed three adverse selection indices in Mudaraba financing. These indices should help financial institutions in reducing adverse selection [18].

In another paper, we tried to test whether the use of a two contract menu can reduce asymmetric information in an environment of incomplete information. We found a game theoretical evidence that Menu contracting is not to be always the optimal option for asymmetric information reduction [19].

In the same line, we have proposed in another paper the offering of an effort based Vs output based contract. In the effort based contract the remuneration of the agent is assessed ex-post based on effort provided. The project financing can continue, even if it fails in the first stage, if the assessment of profit was positive. The second contract reimburses the agent only based on the project output regardless of the effort provided by the agent. This means that the refinancing can only occur if the output is satisfactory. We found a game theoretical evidence that an effort based contract can give higher compensation to the agent as this contract offers a lower sharing ratio to the financier [4]. This result emphasizes two important Islamic concepts. First it emphasizes the sen- timent of altruism which the financier shows by taking a smaller profit sharing ratio. Second it emphasizes the sentiment of positive reciprocity which the agent exhibits by providing high effort [4].

The previous literature discussion treats how to reduce asymmetric information to induce agents to cooperate rather than defect in their relationship. This qualifies PLS to be treated as candidates for a prisoner dilemma case.

It is being argued that the most intuitive way to support mutual cooperation in a repeated Prisoner’s Dilemma game is through reciprocity: A player will cooperate today expecting the other player to cooperate tomorrow [20]

Reciprocal cooperation in Prisoner’s Dilemma game were tested by [21] and [22]. In this test ‘tit-for-tat’ strategy was first in ranking compared to the other strategies in triggering a cooperative behavior. For cooperation to happen in a repeated game we must have a sufficiently high discount factor. In game theory this is known as ‘Folk theorem’ [23]

Even if discount factors are low, the presence of emotions and moral sentiments help individuals balance short-term gains with long-term rewards, as mentioned by [24], [25] claims that players would cooperate if they are confident that the other players also cooperate, but defect otherwise.

In a multi-agent framework it is argued that ‘Everyone would prefer being a coop- erator in a society of cooperators to being a defector in a society of defectors’ [26]. To promote cooperative behavior, therefore, honest agents should provide the example of cooperative behavior to the rest of the population[20].Summary

If players meet each other randomly, then the only evolutionary stable equilibrium is defection [27]. Consequently, tit-for-tat does not become a stable strategy in this environment[20].

The paper is organized as follows:

Section 2 describes the model. Section 3 presents the methodology. Section 4 presents the methodology. Section 5 represents the results and discussion. Section 6 concludes with summary and possible extensions.

2. THE MODEL

The model strives to test for the existence of a prisoner’s dilemma game in a PLS contract between risk neutral bank and a corporation. The later is willing to undertake a project which requires funding F. The later has an initial fund \( f \) but requires an additional funding \( F-f \). The project is estimated to result in a verifiable output \( Y_f \) which is subject to a degree of misreporting \( \Theta \in [0,1] \).

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27 An Islamic term for a form of business in which the bank is the sole provider of capital (Rab’al Mal) and the entrepreneur is the provider of work and management (Mudarib).

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The opportunity cost of the corporation is valued as a ratio $c\%$ of its contribution in the project. The opportunity cost of the bank is $\rho$ i.e. The expected output generated by the project should at least be equal to $(1 + \rho) \beta . \bar{F}$. The expected output given a degree of misreporting $\Theta$ is then:

$$E(\pi_t/\Theta) = \Theta \pi_t = \Theta (1 + R) \beta . \bar{F}$$

(1)

Where “$\pi$” at time “$t$” is the expected return on investment from the project.

The expected output is shared according to a sharing ratio $\alpha$. In case of loss, the maximum loss each partner can lose is his /her contribution in the project capital. For convenience we take $\beta$, as the ratio of capital that is financed by the bank.

The symmetric case

Under this case the bank and the corporation engage in a cooperative agreement. So, the instantaneous expected profit of the bank is:

$$E(\pi^{b}) = [\alpha (1 + R) + (1 + c) (1 - \beta)] F$$

(2)

While the profit of the corporation $E(\pi^{corp})$ is:

$$E(\pi^{corp}) = [(1 - \alpha (1 + R) + (1 + c) (1 - \beta)] F$$

(3)

We turn now to the asymmetric case:

The asymmetric case

In this case neither the bank nor the corporation knows the future strategy of each others. To be able to discriminate the best strategy of each agent vis-à-vis the other agent strategy we use a repeated game theoretical approach supported by an agent based simulation using Netlogo.

The following strategies are set for each agent:

- Cooperate: where each agent is set to cooperate in each round. The bank is cooperating by setting a low profit sharing ratio $\alpha_c$ while the corporation is cooperating by not misreporting the project output.
- Defect: where each agent is set to defect in each round. The bank is defecting by setting a high profit sharing ratio $\alpha_h$ reflecting a misreporting risk premium $R_p$ such that $\alpha_h = \alpha_c (1 + R_p)$ while the corporation is defecting by misreporting the project output.
- Tit-for-tat: each agent responds by the previous strategy of his/ her opponent. For example if one agent cooperates initially, the other responds by cooperating and vice versa.
- Tit-for-two-tats here the agent gives a two chances before changing his /her strategies. For example if the bank cooperate initially and the corporation defect, the bank will cooperate again. If in the second stage the corporation defects, the financier will defect.
- Unforgiving: in this case the agent responds by defecting for ever when the other agent defects once regardless if the later cooperated before.

3. METHODOLOGY

To test for the existence of a prisoners dilemma in a PLS contract. We construct a repeated game in Netlogo using the different strategies of each agent. The following figure shows how the interface of the model looks like:

![Figure 1. The Model interface in Netlogo](image)

We proceed by formalizing the payoff, cumulative payoffs, and average payoffs of each agent under different combination of defection and cooperation:

The Cooperation Cooperates and The Bank Defects

If the bank cooperates while the corporation defects, then the payoff at each round to the bank is:

$$2038892.2$$

$$2987107.8$$

$$3047107.8$$

$$2078892.2$$
While its average profit at each round is:
\[
E_{\text{ave}}(\pi^b) = \frac{\sum_{t=0}^{N} \left[ a_t (1+r) \Theta_t \cdot (1+p) \beta \right]}{N} \tag{6}
\]
While the profit of the corporation is
\[
E(\pi^{corp}) = [(1-\alpha)(1+r) \Theta_c \cdot (1+c)(1-\beta)] \tag{7}
\]

**The Bank Defects and the Corporation Cooperates**

If the bank defects while the corporation cooperates, then the payoff at each round to the bank is:
\[
E(\pi^b) = \left[ a_t (1+r) \cdot (1+p) \beta \right] \tag{10}
\]
Its cumulative profit is
\[
E_{\text{cum}}(\pi^b) = \sum_{t=0}^{N} \left[ a_t (1+r) \cdot (1+p) \beta \right] \tag{11}
\]
While its average profit at each round is
\[
E_{\text{ave}}(\pi^b) = \frac{\sum_{t=0}^{N} \left[ a_t (1+r) \cdot (1+p) \beta \right]}{N} \tag{12}
\]
While the instantaneous profit of the corporation
\[
E(\pi^{corp}) = [(1-\alpha)(1+r)\Theta_c \cdot (1+c)(1-\beta)] \tag{13}
\]

**Both the Bank and the Corporation Defect**

In this case the instantaneous, cumulative and average profit of the bank are given respectively as:
\[
E(\pi^b) = [a_t(1+r) \cdot (1+p) \beta] \tag{16}
\]
\[
E_{\text{cum}}(\pi^b) = \sum_{t=0}^{N} [a_t(1+r) \cdot (1+p) \beta] \tag{17}
\]
\[
E_{\text{ave}}(\pi^b) = \frac{\sum_{t=0}^{N} [a_t(1+r) \cdot (1+p) \beta]}{N} \tag{18}
\]
While in this case the instantaneous, cumulative and average profit of the corporation are given respectively as:
\[
E(\pi^{corp}) = [(1-\alpha)(1+r) \Theta_c \cdot (1+c)(1-\beta)] \tag{19}
\]
\[
E_{\text{cum}}(\pi^{corp}) = \sum_{t=0}^{N} [a_t(1+r) \Theta_c \cdot (1+c)(1-\beta)] \tag{20}
\]
\[
E_{\text{ave}}(\pi^{corp}) = \frac{\sum_{t=0}^{N} [a_t(1+r) \Theta_c \cdot (1+c)(1-\beta)]}{N} \tag{21}
\]

**Both the Bank and the Corporation Cooperate**

In this case the instantaneous, cumulative and average profit of the bank are given respectively as:
\[
E(\pi^b) = [a_t(1+r) \cdot (1+p) \beta] \tag{22}
\]
\[
E_{\text{cum}}(\pi^b) = \sum_{t=0}^{N} [a_t(1+r) \cdot (1+p) \beta] \tag{23}
\]
\[
E_{\text{ave}}(\pi^b) = \frac{\sum_{t=0}^{N} [a_t(1+r) \cdot (1+p) \beta]}{N} \tag{24}
\]
While in this case the instantaneous, cumulative and average profit of the corporation are given respectively as:
\[
E(\pi^{corp}) = [(1-\alpha)(1+r) \Theta_c \cdot (1+c)(1-\beta)] \tag{25}
\]
\[
E_{\text{cum}}(\pi^{corp}) = \sum_{t=0}^{N} [a_t(1+r) \Theta_c \cdot (1+c)(1-\beta)] \tag{26}
\]
\[
E_{\text{ave}}(\pi^{corp}) = \frac{\sum_{t=0}^{N} [a_t(1+r) \Theta_c \cdot (1+c)(1-\beta)]}{N} \tag{27}
\]

The following codes in Netlogo represents the setup of the strategies of each agent:

**Figure 2.** setup of the strategies of each agent in Netlogo

```
if strategy = "cooperate" [ cooperate ]
if strategy = "unforgiving" [ unforgiving ]
if strategy = "tit-for-tat" [ tit-for-tat ]
if strategy = "tit-for-two-tats" [ tit-for-two-tats ]
if strategy = "unforgiving" [ unforgiving ]
```

In the next code we identify how each strategy works:

**Figure 3.** How the strategies of each agent work in Netlogo

```netlogo
if strategy = "random" [ act-randomly ]
if strategy = "cooperate" [ cooperate ]
if strategy = "unforgiving" [ unforgiving ]
if strategy = "tit-for-tat" [ tit-for-tat ]
if strategy = "tit-for-two-tats" [ tit-for-two-tats ]
```

The final step in the coding identifies how each agent payoff is calculated. This include the gross payoff (in Netlogo this is called gross cost) and the net payoff (called score) which takes away the opportunity cost of each agent. Due to its length we leave it to the appendix where we provide the setup and run time procedure.
4. RESULTS AND DISCUSSION

We run a Netlogo simulation using different values of our decision parameters for 1000 runs. Since each period is composed of two consecutive actions, this represents 500 rounds. We take an initial case where \( \alpha = 30\% ; \alpha p=50\% ; \beta = 20\% ; p = 10\% ; c =10\% ; \beta t = 60\% .\)

The findings suggest that no party has an incentive to deviate from their strategies. Any deviation results in a lower (if not negative) profit to one party and a higher income to the opponent. The fact that the bank has developed an unforgiving strategy suggests that the corporation will consider the negative outcomes of deviation and therefore cooperates by the strategy: Tit-for-tats.

5. CONCLUSION

In this research we have tried to reduce the moral hazard problem which results from the corporation defect by misreporting its project results. To do so we have applied a repeated game theoretical approach using Netlogo as multi-agent based modeling interface. In a one stage game we found a simulation evidence that the PLS is not a prisoner’s dilemma case as the two agent are not mutually better off when they cooperate. In a repeated framework, we found simulation evidence that Nash
equilibrium occurs when the bank develops an unforgiving strategy towards defection. Taking that into consideration, the corporation's best response is found to be: Tit-for-tat. Since the bank is assumed to start with a cooperative behavior under the unforgiving strategy then Tit-for-tat, in this case, implies also a cooperative behavior. A future extension of this work is to convert the current form of our PLS contract into a declining balance PLS where the ownership of the project moves gradually from the bank to the corporation. To illicit cooperation of the corporation we propose the transfer of the project’s ownership to be possible via the application of real options.

REFERENCES