TARGETING FIXED REINVESTMENT RATE WITH NONLINEAR MECHANISM: A “STRANGE” PATH TO FINANCIAL DISTRESS

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Abstract

Governance arrangement between shareholders, debtholders and managers fix the reinvestment ratio of profits. Residual earnings will appear as excess cash flow to disgorge in dividend disbursements or share repurchases. However, financial crisis stimulates corporation to express highest aversion both to overinvestment or underinvestment, probably in an identical degree. Besides, dissuasion to commit fraud pushes ownership to select a strong dynamical mechanism adjusting held earnings to the preferred reinvestment rate. Focus? Immediate disbursement of free cash flows. This paper shows that self-imposed discipline targeting fixed reinvestment rate under nonlinear adjustment speed can inject itself a “strange” dynamics to the firm, leading to critical losses and a bankruptcy threat. However, one way to reduce this instability is determining carefully the “normal” cash flow which does not trigger the payout.

Keywords: self-imposed discipline, capitalization, dynamical model

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I. Introduction

A financial dilemma deeply studied in an abundant literature from the seminal paper of Jensen (1986) is the latent conflict between ownership and management about the amount and the destination of the free cash flow (FCF). From a theoretical viewpoint, FCF is a remainder liquidity expelled out of the firm after that level of capitalized earnings was determined ex ante as a strategic target by the corporation. In fact, to avoid underinvestment problem (Mayers and Smith, 1987), and the symmetric risk of over-investment (Richardson, 2006), the management should apply the prevailed and agreed dynamic path of reinvestment. Optimal reinvestment level should emerge as an explicit or implicit agreement from corporate financial governance (Fama, 1974). This is an item of the nexus of contracts building the firm.

In other words, payout stream concerns residual earnings while permanent returns of capital are capitalized as self-financing assets.

Indeed, (extra) dividend and/or repurchase programs are the two mainly ways to insure these disbursement stream (Grullon and Michaely, 2002; Oswald and Young, 2008).

However, self-imposed discipline of fixed reinvestment earnings ratio constitutes governance agreement between ownership and executives to reduce wasted earnings dispatched in inefficient investments but could prevent fraud temptations when free cash flow remains available to the managers (Agrawal and Chadhan, 2006; Agrawal and al., 1999).

This additional corporate governance goal stimulated by multiple accounting scandals is allowed by faster disbursement stream of FCF. A strong speed of payout is reached by selecting nonlinear mechanism of adjustment to retain earnings in relation to the gap with its total amount.

In order to encompass the dynamic implications of the (self) imposed discipline of capitalization via payout of the nonrecurring earnings, we propose a heuristic model of a representative (and hypothetical) firm with cash cows committed to pursue a sustainable reinvestment trend. The model is a modified nonlinear dynamical system of three ordinary differential equations proposed by Bouali (1999) where Profit, Reinvestment, and Financial inflow (debt) are the state variables (Sec. II).

We simulate the dynamics of a firm with a targeted strategy of reinvestment where a nonlinear mechanism releases convergence by disbursements. We investigate the outcome of the management alignment to shareholders’ interests then to debtholders’ interests when the power of the firm is in their hands. The improved mechanism of payout will be revealed when the firm experiences a different calibration of the expected “normal” profit which is detected by its bifurcation diagram to prevent financial hazards (sec. III). We argue that nonlinear payout policy is, itself, the mechanism of fluctuation. The concluding remarks report some implications of our heuristic research and highlights on how automated payout procedures generate costs to the corporation (sec. IV).
II. A Conceptual Framework

We intend to emphasise stylized facts of corporate finance when it self-imposes a mechanism of convergence to a targeted level of capitalized earnings where the disbursements policy is a derived consequence. Obviously, our model describes elementary patterns of the firm and focuses only heuristic simulations of nonlinear adjustments of the convergence. Results are qualitative to a large extend.

We set up a modified version of a nonlinear system model proposed by Bouali (1999 & 2002) in which Profits, Reinvestments and the (external) Financing of the firm’s activity are simultaneously determined. Written in three first-order differential equations, the modelization represents the first principles and rules of the disciplining finance of the modern firms.

Theoretically, in the first equation, the basic premise of the earnings’ determination is allowed by choosing optimal investment. Indeed, the capital allows the creation of profits \( P \) which is made up of Reinvestments and financed also by a capital inflow, i.e. the debts \( F \).

\[
\frac{dP}{dt} = v (R + F) \tag{1}
\]

\( v \) rate of profits.

On the other hand, when the incentives to shareholder’s underinvestment are reduced, the managers encourage reinvestment which expands the production capacity of the firm and enhances the shares value and avoids its dispersion. Reinvestment constitutes an important item of the global reliance of the corporate governance. We notice that supplemental investments have identical profitability (the scalar \( v \)) of the previous projects.

On the other hands, in a recent survey of the motives of the payout policy by Brav and al. (2005), operational and investment choices are made before dividend payout or stock repurchases. In concordance with the investment prevalence, firstly the cash spent on capital acquisitions or mergers can be determined 

\[
\frac{dR}{dt} = m P \tag{2}
\]

where \( m \), the reinvested earnings ratio.

Meanwhile, interrelated to this targeted reinvestment in positive NPV projects, the payout starts to reduce potential overinvestment if extra cash is earned. In this direction, when the profit reaches the anticipated “normal” value (by the corporate management) \( P = 1 \) monetary unit (m.u.), \( m \) becomes the amounts of reinvestments and no dividends or stock repurchases are featured. “Normal” value of the expected Profits is necessarily a “normative” amount determined by an explicit or implicit evaluation from the corporate governance. Facing to multiple hypothesis of growth, “normal” value of profit chosen by corporate arrangement is a composition from pessimistic and optimistic viewpoints. This is the key of both capitalization and payout processes.

On the contrary, if \( P \neq 1 \), it triggers off a mechanism of convergence to the selected level of capitalized earnings and the disbursement of the nonrecurring surplus cash (Bagwell and Shoven,1989; Lie, 2000) or symmetrically holding more earnings. As, the complete equation becomes:

\[
\frac{dR}{dt} = m P + (P^* - P) n R \tag{2}
\]

where \( P^* \), amount of “normal” earnings which do not release any adjustment process.

Indeed for \( P = P^* = 1 \), the reinvestments trend takes the targeted \( m \) value and no procedures of payout are launched. However, in case of losses reaching value \( P = -1 \), the firm initiates a divestitures at the trend: - \( m \).

Moreover, the nonlinearity allows a strong payout when Profits exceed \( P^* \) and a decrease of the capitalization of earnings valued at the rate \( n \). Beyond what a firm could invest, extra funds are strongly reduced to reach the \( m \) ratio of self-investment by the intensification of disbursements, or the more flexible stock repurchases, according to the gap between 1 and \( P^* \).

Similar specification reveals lack of trust on corporate governance since this payout procedure implies an immediate extraction of the FCF from the manager’s hand with accelerated speed.

Symmetrically, when the mass of Profits is lower to \( P^* \), earnings are capitalized with a fast increase. Nonlinear item arises strongly and pushes management to reduce payout to compensate the lack of profits. To prevent financial distress and underinvestment threat, Payout is braked since cash flow shortage is a critical phenomenon (Uhrig-Homburg, 2005). Capitalization must grow at a strong rate to converge to the \( m \) value and the stock buybacks, or the dividend payout, is decelerated.

When \( P < P^* \), the Reinvestment Privilege or the automatic reinvestment of shareholders’ dividends in more shares by a Dividend Reinvestment Plan (or 

stock dividend) is used. The convergence pattern will keep funds to self-finance the capital assets. Besides, for the weak amounts of profits, the firm must resort to financing reinvestments by self-tender offers of new equities or shares’ issuances into the open market. However, the firm divestitures its capital assets when accumulates losses.

The regulation’s mechanism and its specification violate neither the “orthodox” behaviour of the managers nor the principles and rules of the disciplining practice of finance governance. In fact, the aim of the mechanism is the driving of Reinvestment to desired level \( m \) and hinders the retention of excess liquid assets since that FCF, or its shortcoming, constitutes the hidden parameter of the second equation. \( P^* \) is a threshold which separates between supplemental reinvestments and triggering off payout.

It is worth noticing that equations encompass the payout stream with a simple pattern: only the profit’ driver is the investment process.

In the actual international economic context of financial crisis and fraud scandals, a strong level of a monitoring activity is chosen introducing managerial inertia as a new agency cost. Indeed, in our model, even if identical profitable NPV investments are
available, the firm pursue the prevailed governance arrangement of reinvestment rate (and its stabilization automaton) until the next managers’, shareholders’, bondholders’ deliberations.

Eventually, the third equation is the account of the net capital inflow of the firm:
\[
\frac{dF}{dt} = -rP + sR \tag{3}
\]

After deducting the capital outflow (the debt service ratio), the corporate borrowing is obtained according to the debt/equity ratio \(s\). In fact, the debt service is linked to the volume of loans but for ease of the simulations, our basic formulation simplifies the model and does not modify fundamentally the core of the corporate governance.

Simulations of the model serve to check the implications of the imposed (or self-imposed) discipline of capitalization policy when nonlinear mechanism of convergences is made.

III. Computational Results

The basic study begins with the detection of the solutions of the system:

\[
\begin{align*}
\frac{dP}{dt} &= v(R + F) \tag{1} \\
\frac{dR}{dt} &= mP + (P^*-P) nR \tag{2} \\
\frac{dF}{dt} &= -rP + sR \tag{3}
\end{align*}
\]

All variables are endogenous and the steady-state equilibria are obtained for \(dP/dt = dR/dt = dF/dt = 0\). We get \(F = -R\) from (1), \(n(P^* - P)R = mP\) from (2) and \(P = sR/r\) from (3). The last two relations yielded the following equality: \([(P^2 - P^*) n rP/s] - mP = 0\).

The three roots of \(P\) are: \(P_1 = 0\), \(P_2 = [(ms/nr) + P^*/2] \) and \(P_3 = -P_2\). Let \([(ms/nr) + P^*/2] = k\), the three equilibria become: \(E_1(P, R, F) = (0, 0, 0)\), \(E_2(P, R, F) = (k, rk/s, -rk/s)\) and the third solution \(E_3(P, R, F) = -E_2\). Jacobian matrix of the 3D system gives \(|J| = v[nP^2 + ms - 2nPRs]\).

Numerical computations are carried out with the fifth-order Runge-Kutta integration method and \(10^{-9}\) accuracy and the initial conditions are \(IC(P_0, R_0, F_0) = (0.01, 0.01, 0.01)\).

We select a set of parameters as the financial statements of the firm \(C(v, m, n, r, s) = (0.25, 0.04, 0.02, 0.1, 0.3)\) and the expected “normal” profits: \(P^* = 1\).

The trajectory of the system (Fig. 1) follows an infinite orbit centred on the equilibrium: \(E_2(P, R, F) = (2.64, 0.88, -0.88)\). The firm as a dynamical system oscillates without any periodicity in the phase portrait of the state variables, profits, reinvestments and capital inflow [1].

The dynamic of the firm fluctuates chaotically between weak negative values and high levels of profits \(P\). The attractor centered around \(E_2\) is displayed when initial conditions \(IC(R_0, P_0, F_0)\) are positive. With negative IC, the simulation displays an anti-symmetric chaotic attractor centered around \(E_3\) which is the result of Sensitive Dependency on Initial Conditions (SDIC).

![Fig. 1. Chaotic attractor for \(P^* = 1\)](image)

Theoretically, if and only if the initial conditions are \(E_2\), the steady-state is obtained. In practice, the \(E_2\) values cannot be attained with an infinite accuracy since a very weak lag pushes the trajectory farther from the equilibrium. Even if recorded financial data to built projections, analysts’ forecasts or extrapolation of historical data have \(10^{-5}\) accuracy, they do not allow reconstruction of the “real” model since the missed ten-thousandth fraction of the variables hinders the perfect estimation of the current financial statement.
Paradoxically, the chaotic attractor as a dynamical object confined in a limited set of the phase portrait prevents predictions of the variable values and reflects an infinite number of dynamical periodicities. At the most fundamental level, the deterministic nonlinear mechanism in Eq. 2 can imply chaotic motion of the state variables of the firm. Triggering off sequences of reinvestment-divestiture procedures marks persistent and non-transitory chaotic oscillations and a failed management practice leading to other governance arrangements. In fact, the management of public firms can be a subtle balance (and neutralisation) of the stockholders and bondholders interests allowing to wide autonomous managerial actions with a minimum level of monitoring interference. Inefficient financial policy or unexpected and unpredictable instability yields to the lost of the management autonomy and leads sometimes to a “big bath” (a supplemental agency cost!) requested by both groups of interests. Meanwhile, unbalanced interests could put decision management rights and decision control rights introduced by Fama and Jensen (1983), in the same hands of this or that group of holders which invades the operational field (Fluck, 1999). The instability leads to the adoption of an alternative governance arrangement substituting this failed management policy (Bhagat and Bolton, 2008).

III.1. Alignment of payout policy to debtholders interests

If bondholders monitor the firm, they enforce their directives into the management. The power of decision (Rajan and Zingales, 1998) is now in their hands and they could postpone payout focusing on the extinction of instability.

In our application, the managers might decide to experience a new speed of adjustment and, simultaneously, define other “normal” earnings. The management can choose a different calibration of $P^*$ following objectives and beliefs of debtholders to avoid hazards of chaotic motion incurred by the previous convergence procedure. For example, the new payout strategy delays earning’s distribution with $P^* = 1.8$. Therefore, triggering the disbursement decision beyond a high threshold of cash flow thereby allows a resource which will boost the reinvestment rate and guides, in first principle, to a sustainable trend of growth. Indeed, zero-payout is yielded [2] for the new strategic reinvestments raising from $\frac{dR}{dt} = 0.04$ up to 0.05.

The approach which governs the new strategy is focused not on the present earning’s distribution but on the future creation of the profits. Moreover, postponing realization of capital gains through dividends allows the investors a preference to the “timing tax option” (Constantinides, 1984), whose taxation of several annual earnings is less than that of the quarterly frequency tax.

Fig. 2. Chaotic attractor for $P^* = 1.8$

The deterministic trajectory drives the firm to profitability and loss without any prediction. The basin of the chaotic attractor expanded to its anti-symmetric set leads to a wide array of losses. The unstable equilibria are $E_2(R, P, F) = (2.79, 0.93, -0.93)$, $E_1(R, P, F) = (0, 0, 0)$, and $E_3 = -E_2$. Changing $P^*$ from 1 to 1.8, the system displays the Sensitive Dependency on Parameters (SDP) of the chaotic system. Associated with the SDIC, the chaos of the dynamical system becomes strange.
Surprisingly, and contrarily to the expectations, the behaviour of the state variables is projected to a basin of attraction where the profit P takes a wide range of gains but also a wide set of losses (Fig. 2). Keeping cumulative surplus of liquid assets triggers disbursements of their squared amounts which are financed by earnings, equity issues, and also by debts.

What is the optimal P* leading to the minimum of profit’s instability? In fact, simulating the 3D system for a set of P* allows the detection of the dynamics of P and plots the diagram of bifurcation (Fig. 3).

Against the orthodox principles of management, incremental P* guides the profits to an expanded chaotic bubble.

Risk of negative profitability rises sharply beyond P* ≈ 1.5 since the possible worst performance of P moves from -2 to -6 despite postponing earning’s distribution. The deterministic chaos vanishes when P* is selected in the stability windows of the bifurcation diagram.

III.2. Alignment of payout policy to equityholders interests

If insider ownership and the block of common shareholders inspire and monitor management

(Schleifer and Vishny, 1986; Blair, 1995) they obviously evaluate “normal” earnings P* at a lower value. Triggering early payout stream even in case of weak profits reflects the management’s alignment to shareholders interests in the earning’s distribution since they constitute the powerful group of the firm.

If P* = 0.4, targeted reinvestments fall and zero-payout is yielded to a low motion: dR/dt = 0.02 [3]. The dynamical trajectory of the corporation follows period-4 orbit (Fig. 4). Efficient cash management is not only putting cash to applications more quickly to produce earnings but also exiting excess cash flows to reduce the periodicity of capital assets.
The state variables oscillate around the unstable equilibrium \( E_2(R, P, F) = (2.53, 0.84, -0.84) \) in the simplest orbit reachable with a perfect and predictable recurrence if the equityholders impose a reduction of the reinvestment stream and an early payout triggering-off. The other equilibria are also unstable \( E_1(R, P, F) = (0, 0, 0) \) and \( E_3 = -E_2 \).

The result is consistent with high legal protection of shareholders which allows the increase of dividends even in case of low profitability (Laporta and al., 2000).

Whether the payout mechanism injects oscillations for all values of \( P^* \), a low periodicity of the firm’s variables is obtained when disbursements are released from very weak amounts of cash but with a relative underinvestment. We notice that modification of \( P^* \) displays the morphological plasticity of the attractors and demonstrates the Sensitive Dependency on Parameters (SDP) of the 3D system. The wide range of patterns is obtained with only one nonlinear equation.

IV. Concluding Remarks

Corporate governance has now reached a level of sophistication far beyond our idealized numerical experiments. Yet, our three dimensional model of the firm where the disbursement policy is an implicit variable serves only as a heuristic tool to detect the implications of a mechanism of convergence to a targeted capitalization rate. It reflects well and rational choices of the corporate management art.

To our knowledge, this model is the first attempt to study dynamical findings of self disciplining profit’s capitalization in the context of deep lack of trust between ownership and control. Numerical computations are carried out also when the power is transferred from shareholders to debtholders.

In our application, the nonlinear regulation of the reinvestment is investigated with the tools of the theory of deterministic chaos (Baumol and Benhabib, 1989; Day, 1994) which can complete the recently established framework of econophysics (Mantegna and Stanley, 1999).

Singular results are made in opposition to rules and principles of finance governance built in static and linear framework since lead to a chaotic dynamics and strange attractors (Bouali, idem, 1999). Firm can lose its dynamic stability when targets fixed reinvestment stream. The gap between actual earnings and their expected amount releases the mechanics of payout but automated financial governance procedures imply costs.

Indeed, the main findings of our nonlinear and ab initio heuristic model show the negative implications of a self-imposed discipline of disbursements when the outflow earnings are erroneously triggered (amounts and frequency).

Committing to pay out excess of earnings resolves an agency conflict, but, it can also inject fluctuation leading to a chaotic hazard and bankruptcy threat added to the wider array of identified financial risks.

For example, postponing disbursements of excess cash is not harmless and can introduce a critical dynamic motion. The normal earning’s threshold \( P^* \) which drives capitalization should be technically determined, for example, by a bifurcation diagram. \( P^* \) is a key parameter of the corporation derived from the size and the profitability gathered in the financial statements parameters \( C(v, n, n, r, s) \).

Meanwhile, if the payout device maintains the interests of equityholders or bondholders, its
automated mechanism plays against their interests themselves! The particular specification of the linkage between reinvestment target and payout policy inserts a chaotic dimension, or almost a periodicity into the profit.

Assuming the simplest formulation of the model, the self-financing procedure to focus level $m$ is, itself, the turbulent process and not a transitory phenomenon. In a few words, the dispatching of the profit (to reinvestment and payout flows) modifies the dynamic stability of the profit itself.

The first insight from our application can be seen as disciplining profit capitalization policy is, itself, a mechanism of fluctuation. Oscillations are not an artefact yielded by the simulations but the outcome of the nonlinear disbursements behaviour. Payout mechanism has an oscillatory nature since our heuristic system without the second item of Eq. (2) leads to an exponential growth of $P$. Our outlook is consistent with Baker and Smith (2006) conclusions. They indicate that some firms “…may follow a “modified” instead of “pure” residual dividend policy to avoid highly volatile dividend payments.” Intuitively, managers “disconnect” the payout’s automaton and drive “manually” the earnings’ disbursement to pull backward the system far from the chaotic bubble.

We could argue that payout automaton generates an endogenous, singular and deterministic financial distress which is not driven by incomplete, imperfect market considerations or industry arguments.

Our 3D system is consistent with the objectives of bondholders if they enforce their interests to the management since delaying payout policy generates an overinvestment. Likewise, when stockholders inspire early disbursements, the system leads to underinvestment. In fact, an imposed payout requires a somewhat fine-tuning application to control oscillations: a small disbursement chain (low value of $P^*$) braking the chaotic expansion of the profit variable. It is a new justification of the “reluctance” to cut dividends (Kalay, 1980; Frankfurter and Wood, 2003).

In our application, payout is not the residual of reinvestments but is triggered out by the nonlinear adjustment of capital reflecting high sensitivity of ownership to the cash emergence. The strong elasticity of payout to earnings can be a consequence of severe agency conflicts, and derived from scarce level of trust between stockholders and debtholders and managers (Farber, 2005).

The payout’s simulation approach can be further applied to several sets of parameters particularly $m$, which is derived from the expected earnings stream. A more promising path for investigating the consistency of the present conclusions is carrying out simulations with other kinds of reinvestment convergence.

Deep insights in the shape of these regulators and their dynamical implications could enhance the theory of corporate finance (Dühnfort and al., 2008; Schleifer and Vishny, 1997), and its reliability.

However, our basic model of the financial statements marks a loss of generality and deserves a sophisticated formulation. For example, managers perceive a substantial asymmetry between dividend cut’s decision and its increase (Brav, 2005, idem) and, also, between financial distress and profitability. The nonlinearity in Eq. (2) is adapted only to a perfect symmetry of the flows. Meanwhile, fixing ex ante a level of $m$ denying per se investments in unexpected NPV projects and hindering fund conversion in profitable assets is a credible agency cost of corporate finance.

Moreover, in case of loss, the opposite motion emerges since the reinvestments are converted into divestitures in Eq. (1) and the borrowing inflow to an outflow in Eq. (3).

Eventually, this study contributes to the controversial debate of the dividend policy initiated by the seminal paper of Miller and Modigliani (1961). A New evidence that payout policy mechanism is not “irrelevant” to the stockholders wealth or the firm’s valuation.

References


Endnotes

1. The other equilibria are $E_1(P, R, F) = (0, 0, 0)$ and $E_3 = -E_2$. Since $|J|$ is positive, all the equilibria are unstable. The model is conservative for the trajectories that are close to $E_1$ and dissipative particularly at the neighbourhood of $E_2$ and $E_3$.

2. In the case of the loss $P = -1.34$, divestitures stream reaches rate $dR/dt = -0.0536$.

3. In the case of the loss $P = -0.632$, divestitures are $dR/dt = -0.025$. 